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A NOTE ON EXTERNALITY BENEFITS AND THE FEASIBILITY OF PARETO OPTIMALITY THROUGH UNILATERAL SUBSIDIES

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It has long been accepted that consumption (or production) activities which create external benefits to other parties will typically be operated at a sub-optimal level and that subsidies may be used to achieve optimality. The present analysis will deal with an external benefit where the externality is reciprocal between the consumption activities of two individuals and where the benefits are non-rivalrous.¹ A subsidy system may be designed to alter the price of the consumption good so that consumption will be raised to an optimal level. This note will examine the importance of the subsidy transaction and will point out how the number of participants is relevant to the analysis. In particular it will deal with the significance of "third party subsidies".

The analysis deals with two consumers, A and B, who consume some good R. The consumption level of R by A will be designated by R_A and R_A will enter, as a "public good", into the utility functions of both A and B. Consumer A purchases R in the amount R_A and this amount then becomes equally available to both A and B. By the same token R_B represents purchases of R by B and R_B also enters into the utility functions of both A and B. Hence, what we have is a case of reciprocal externality.

I Initial Corner Solution

Figure 1 represents this situation of reciprocal externalities where the amount of the public good equally available to both A and B is measured along the horizontal axis while the income that may be potentially spent on other goods is shown on the vertical axis. The feasible basket of private and public goods available to each of A and B independently is shown by $DO_A S$ and $DO_B S$, respectively. The intervals

$O_A D$ and $O_B D$ measure the income levels of the two consumers, whose consumption levels are measured North-east from O_A and South-east from O_B . The indifference maps of both consumers are assumed to be strictly convex and to exhibit diminishing marginal utility of income.

Consider first a situation where consumer B makes the independent choice of a corner solution at D, in the sense that $R_B = 0$ for $R_A \geq 0$. Figure 1 illustrates this by a price consumption curve DF_B which passes through the budget line at $R_B = 0$. Consumer A, on the other hand, must buy R at the same price $\left(\frac{DO_A}{O_A S} = \frac{DO_B}{O_B S} \right)$ as does B but A places a higher valuation on R and hence purchases R'_A . His price consumption curve DF_A passes through his budget line at H, where $R = R'_A$. Clearly, this will place B on a higher indifference curve than that associated with point D. However, by paying a subsidy to consumer A it is possible for B to improve his welfare position further, along with that of A.

To illustrate this let us first draw DC parallel to the R_A axis in Figure 1. If we measure upward from DC then the ordinate of DF_B gives the amount of money that consumer B is willing to exchange for given amount of R. For example, at J he is willing to give JE in order to obtain DJ of R and this reflects a willingness to pay a price of $\frac{EJ}{DJ}$ for DJ. If we now add the ordinate values of DF_B to those of DF_A by a vertical summation southward from DC then we can derive a joint offer curve DF_{A+B} . This joint offer curve represents a joint willingness on the part of A and B to pay for R_A . The new joint optimum is found at K, where the joint offer curve passes through the price line DS. At K the joint consumption level of A and B will be optimum since the summed marginal rates of substitution will be equal to the price of R_A as may

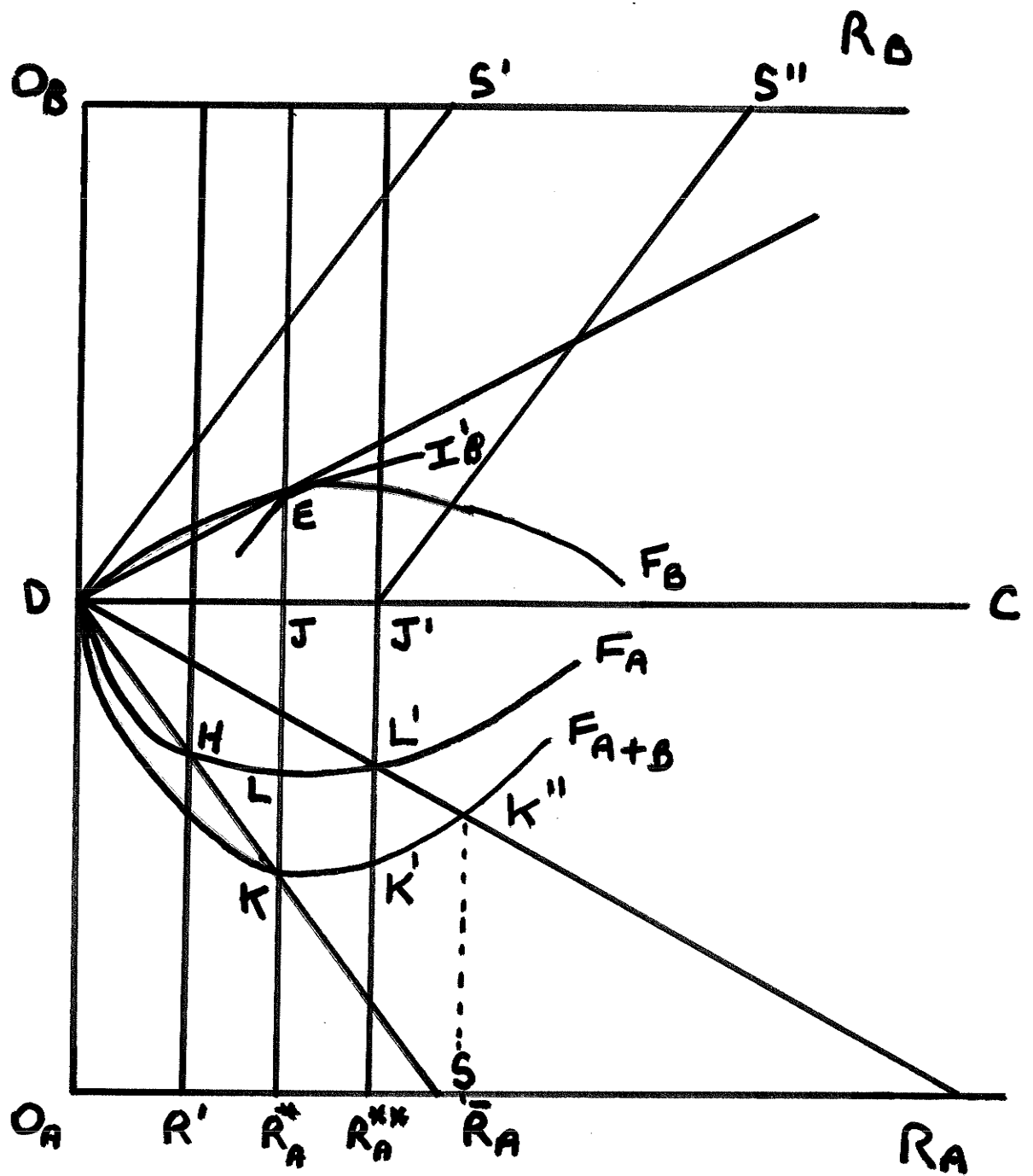


Figure 1.

easily be shown. The ratio $\frac{EJ}{DJ}$ gives the slope of the indifference curve I'_B at E while $\frac{JL}{DJ}$ is the slope of the indifference curve passing through point L. As a result $\frac{JK}{DJ}$ is the combined slope of these two indifference curves and this is equal to the price of R. This suggests that a subsidy of EJ which is paid by B to A will enable Pareto optimality to be achieved.²

The equilibrium positions at points E and L, which are associated with point K, are representative of a set of Pareto optimal solutions. This particular case is one in which the externally affected party compensates the party generating the spillover benefit and where the compensation takes the form of a constant per unit subsidy. The writer has elsewhere (Vardy, 1971b) dealt with compensation schemes in which the per unit subsidy is variable. Let us consider now what happens, remaining within the framework of linear subsidies, when some third party pays the subsidy on behalf of consumer B and at zero cost to B. The "third party" may be thought of as the government. When the government attempts to achieve optimality by subsidizing A without taxing B then this will be referred to as a "unilateral" solution. When the government taxes B to subsidize A or when A compensates B directly this will be called a "bilateral" solution.

Referring again to Figure 1 it will be clear that if A is paid a subsidy of $\frac{EJ}{DJ} = \frac{LK}{DJ}$ then consumer A will purchase R_A^* and this will be optimal if B is paying EJ. When a unilateral solution is attempted, the subsidy will place A at L but it will displace consumer B to a position at J. Since it is assumed that the marginal utility of income is declining then the indifference curve passing through point J will

be steeper than that at E. Consequently, the positions of consumer equilibrium at L and J will no longer be optimal, since the sum of the marginal rates of substitution will exceed the price of R. Hence, R_A^* will be suboptimal and an expansion of R to R_A^{**} will be optimal. A somewhat larger unilateral subsidy will be required in order to achieve R_A^{**} . Let the optimizing subsidy become L'K' and the new positions of equilibrium L' and J' so that the summed marginal rates of substitution are equal to the price of the good. At point J' consumer B's equilibrium will be an inequality solution³ since the indifference curve passing through J' is flatter than the budget line J'S". Hence, neither A nor B will have any incentive to adjust his own direct purchase of public or private goods. However, if bilateral bargaining is possible then it will now be advantageous for consumer B to pay a subsidy to A to induce an increase of R_A to \bar{R}_A at K". At this level of joint consumption the summed marginal rates of substitution will be equated with the price of R_A less the government subsidy. The result of this bargaining process will thus be overconsumption.⁴

This suggests that the payment of government subsidies on a unilateral basis may lead to overconsumption. This is particularly likely when the number of parties involved is small, as in our example. In a situation with a large number of participants the bargaining costs of making bilateral contracts and direct compensation may preclude overconsumption. In fact, it is precisely in the large number problem, where bargaining costs are high, that government subsidies are likely to be necessary.

II Both Parties Initially Consuming Joint Good

The previous section dealt with a public consumption good where only one of the two consumers would independently purchase the good, in isolation from other consumers. This section will deal with the case where both consumers begin from initial interior solutions. As Williams (1966) and Pauly (1970) have shown, two consumers who purchase a good which is reciprocally non-rivalrous will find it advantageous to achieve some mutual adjustment in their purchases.⁵

Figure 2 presents a situation in which the two consumers would choose independently to purchase R_A^I and R_B^I of good R. Assume that A first purchases R_A^I and that B is originally at D. Consumer B now selects position E_1 by choosing to purchase the basket of private and public goods given by E_1' . This has the effect of shifting A's budget line out to D'T so that A now selects position L_1 . This process of reactive adjustment continues until an equilibrium is achieved at R^E with A purchasing R_A^E and B purchasing R_B^E . In general, it can be shown that the amount a person purchases in mutual adjustment equilibrium will depend upon his income elasticity of demand and upon the amount that he would have purchased independently.⁶ Note that the new equilibrium positions lie on the income consumption curves ($O_A Y_A$ and $O_B Y_B$) of both consumers. Hence, the marginal rate of substitution of each consumer must be equal to the price. This means that consumption must be sub-optimal since optimality requires that the summed marginal rates of substitution be equated with price.

The reactive adjustment equilibrium of Figure 2 will be replicated by positions A and B of Figure 3. The curves AF_A and BF_B are the

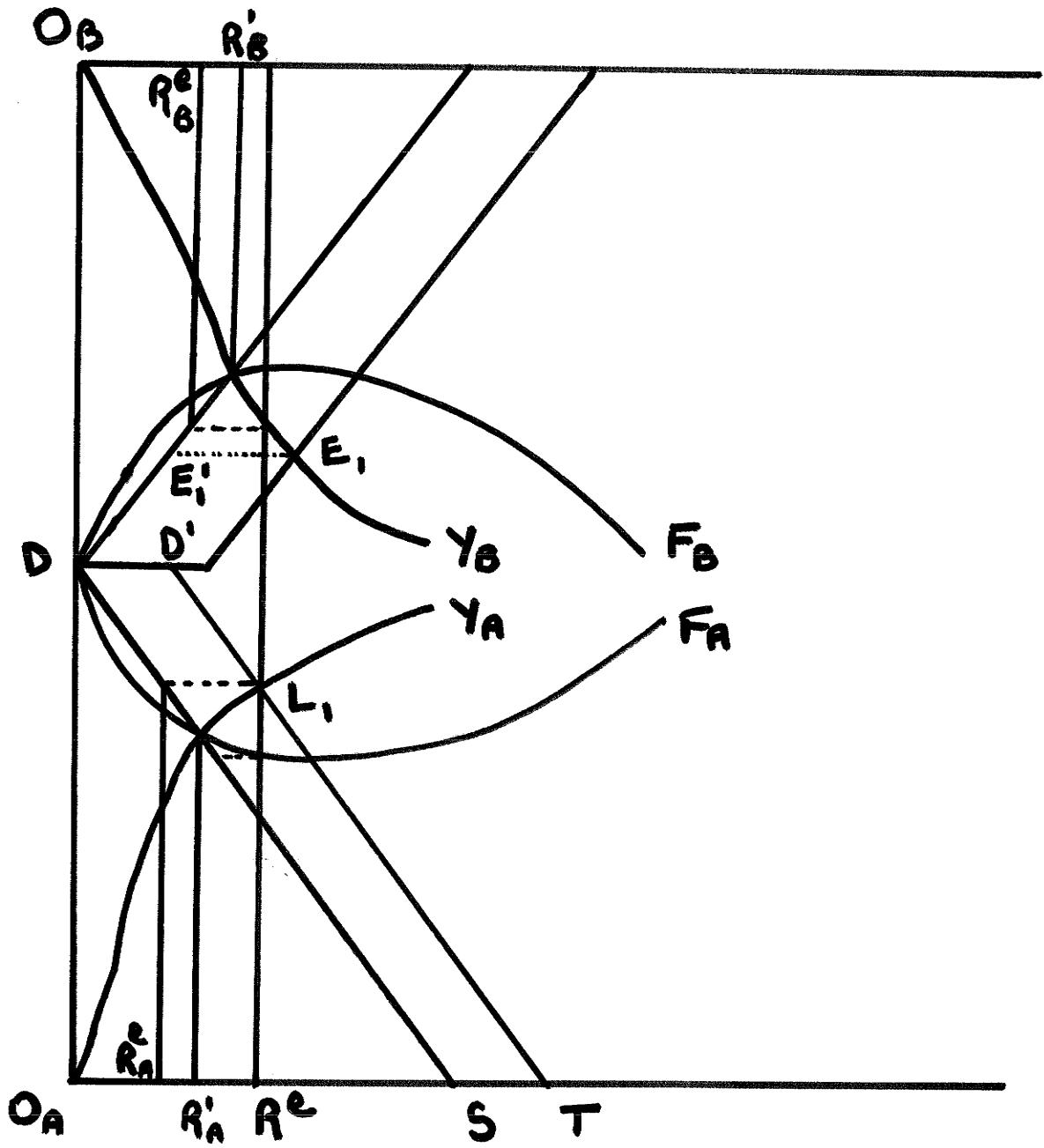


Figure 2.

price consumption, or offer, curves of consumers A and B respectively. If we add BF_B to AF_A , as in the preceding section, then we can determine the optimal joint consumption level from the intersection of the joint offer curve AF_{A+B} with the budget line $D'T'$. The optimal consumption level R^* is based upon the payment of a constant per unit subsidy from B to A for purchases of R by consumer A beyond R_A^e . This effectively makes the budget line facing consumer A $D'AA'$.

If a unilateral subsidy (again on a constant per unit basis) is paid to A by the government and if this shifts the budget line facing consumer A out to $D'AA'$ then B's new position will become B'' . Since B's indifference curve at B'' will be steeper than that at B' (due to diminishing marginal utility of income) it will be found that R_A^* is sub-optimal when the subsidy is paid to A at no cost to consumer B. The summed marginal rates of substitution for the two consumers, corresponding to points A' and B , are equal to the price of R but the sum of the MRS's at A' and B'' will exceed the price. This implies that a larger unilateral subsidy will be required to achieve optimality at R^{**} . The optimizing subsidy corresponding to R^{**} will be $A''C'$ (as compared with $A'C$ for R^*). The Pareto optimal level of joint consumption at R^{**} , corresponding to a unilateral subsidy to consumer A, will place consumer B in equilibrium at point B''' . At this point he will have no incentive to alter his own direct purchases of R. As before, however, there will be an incentive for him to bribe A to consume more units of R, provided that bargaining costs are sufficiently low. In a situation where there are a large number of participants the transaction costs of bilateral bargaining and compensation may be prohibitive so that unilateral solutions

will be efficient. The efficiency of unilateral subsidies will depend crucially upon the parametric role of prices and the incorporation of government subsidies into these price parameters, as well as upon the economies of scale in bargaining achieved by government subsidies. In a small number situation it is unlikely that unilateral government subsidies will achieve optimality, because the participants may attempt to readjust their joint behaviour with respect to the new prices. But when bargaining costs are significant then the solution achieved by unilateral government subsidy will not be vitiated by direct bargaining between participants.

III Conclusion

The purpose of this note has been to show that in the presence of external benefits, both bilateral and unilateral subsidies may be efficient routes toward Pareto optimality. One of the issues involved here is that of the distribution of the costs of the joint consumption good. The smaller the share of the subsidy that is paid by the external beneficiary the larger will be the Pareto optimal level of output. The second issue relates to the number of participants involved, as both externality generators and recipients. If there are few participants then bilateral solutions will be most effective while a unilateral subsidy may lead to overconsumption. The case for unilateral subsidies will be strongest when there are many participants, so that Pareto optimal solutions will not be overruled by direct negotiation and compensation between affected parties.

FOOTNOTES

1. The case of reciprocal and non-rivalrous external benefits is treated by Pauly (1970).
2. The criterion of optimality here is given in terms of the usual first-order Samuelson condition for public goods (Samuelson, 1954).
3. In the sense that the slope of the budget line will exceed the marginal rate of substitution.
4. This corresponds with the problem of unilateral subsidy-cum-bargaining treated by Turvey (1963) where the optimality of the unilateral solution is destroyed by bargaining. Turvey also deals with a two-person case where bargaining costs are low. Also, see Buchanan and Stubblebine (1962).
5. The "mutual adjustment" that is referred to here is the adjustment of consumption from own purchases without explicit bargaining or compensation.
6. To show this, consider Figure 4, where R_A and R_B refer to the purchases of consumers A and B. The purchase line P_A refers to A's purchases of R, corresponding to given purchases by B, while C_A refers to A's consumption of R, corresponding to given purchases by B and A. The curves P_B and C_B are similarly defined for consumer B.

The reaction adjustment equilibrium occurs at E, and the total purchases at $E (= R_A^E + R_B^E)$ are equal to R^E in Figure 2. (Note that the ordinate of C_A is given by the corresponding ordinate of P_A plus the abscissa value of R_B .) If A's income elasticity of demand for R were higher, as shown by C_A' then R_A^E would be higher and R_B^E lower.

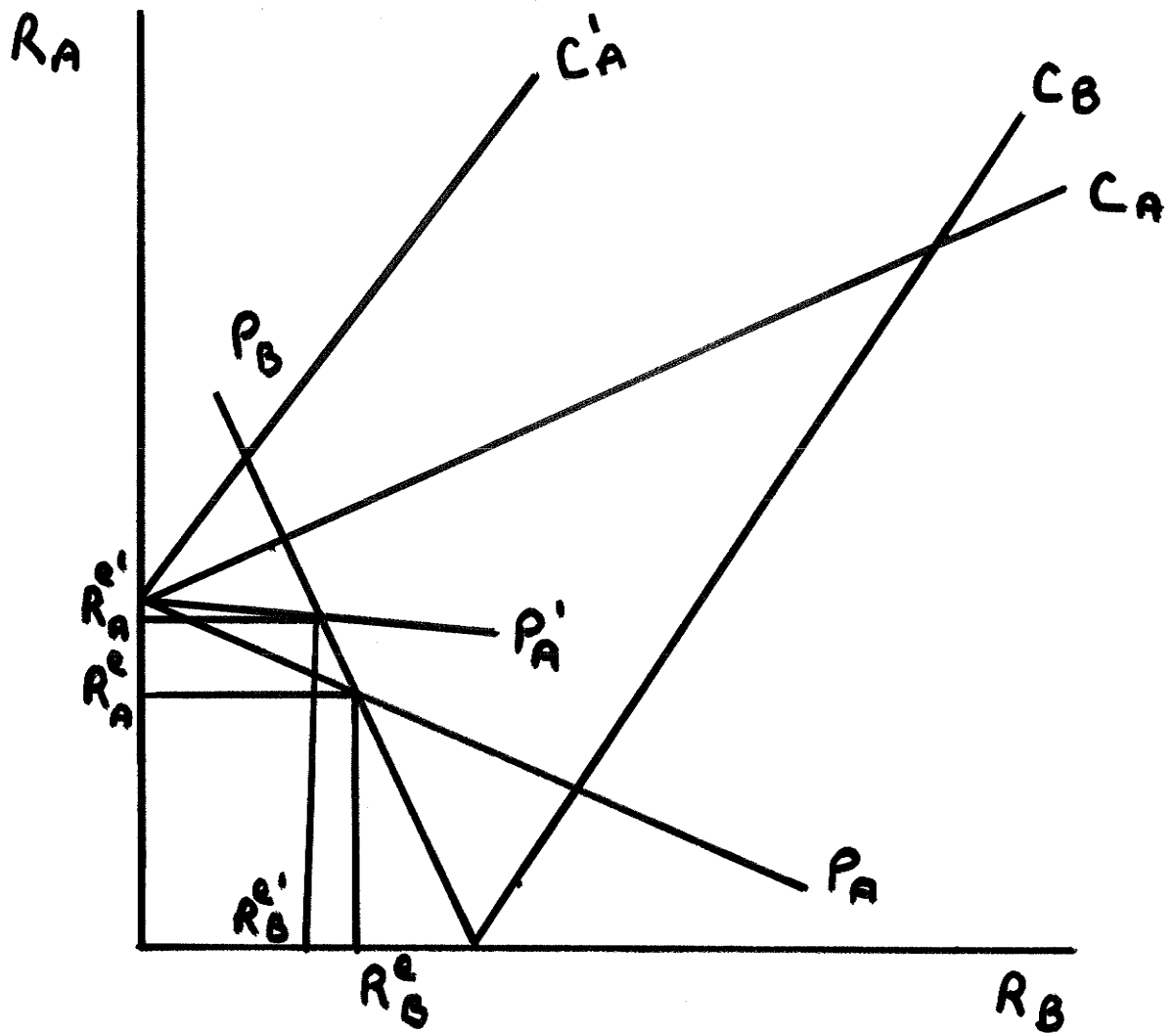


Figure 4.

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